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AN ANALYSIS OF THE GAS FLOW FROM A VERY HIGH PRESSURE NOZZLE

BY

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PROJECT TS4-4014

PITMAN-DUNN LABORATORIES
FRANKFORD ARSENAL
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OBJECT

Studies toward the establishment of a theory of recoilless rifle blast.

SUMMARY

The study of the blast from recoilless rifles requires a theoretical analysis as a preliminary step to an extensive experimental mapping of the blast field. To this end, a theoretical analysis of the flow from a nozzle has been made. A single, wedge shaped nozzle with 7° half-angle of divergence was considered. The ratio of exit height to entrance height of the nozzle is $\sqrt{2}:1$. Chamber pressure and temperature have been assumed to be 5400 psi and 2400° K, respectively. The principal feature of the flow is a concentrated, almost stationary, region of strong shock called the Mach disk. This was not located in the course of the usual calculations because of the limited assumptions of the theory. On the basis of the growth of turbulence in the boundary layer, the distance of the Mach disk from the exit section of the nozzle was found to be between 40 and 140 exit diameters, the first value being the more probable. The pressure and temperature after the shock are 40 psi and 2300° K in the first case, 14.6 psi and 3100° K in the second case. Mach numbers as high as ten occur in the flow. A detailed map of the flow up to a point 140 exit diameters away from the exit section is given. This map should provide a guide for the study of the blast from recoilless rifles, since it enables one to interpret better the results of experimental studies. Furthermore, the location of the Mach disk can serve as a criterion for further improvements in the theory.

AUTHORIZATION

OO 121.2/1695, FA 472/2370, 22 Oct 51

INTRODUCTION

The study of the blast from recoilless rifles must proceed in two parallel modes, the experimental and the theoretical. The first step in the theoretical analysis of the blast of gas from recoilless rifles is the study of flow from a very high pressure nozzle (chamber pressure greater than 1000 psi). This report gives the first approximation in the study of the free gas jet coming from such a nozzle. Such a study is only the first of a series of calculations, but by itself it yields significant information about high pressure jets in general, and thus may be useful as a guide in long range planning.

It is a general feature of high pressure jets that they are separated into two distinct regions by a very strong shock perpendicular to the axis of the nozzle, called the principal Mach disk. Studies of the region downstream (i.e., the region farthest from the nozzle) of this shock may be done independently of work on the upstream side.

This report covers the region of flow (in the two-dimensional case) between the nozzle and the principal Mach disk. The chief results of this work are contained in the maps of the flow field given in Figures 1 and 2 and Table I. These give approximate values of pressure and velocity at any point in the indicated region of gas flow. Preparations are being made to correlate the theoretical analysis of the blast with an experimental map of the flow field.

BASIC DEFINITIONS

The specialized vocabulary of fluid dynamics requires a set of definitions for the full understanding of any report on some phase of that subject.

Sonic Velocity

The velocity of sound in the medium at the point under consideration; usually assumed to be the square root of the ratio of the specific heats of the gas times the local pressure divided by the local density.

Hodograph

Refers to the "velocity" plane, the plane whose coordinates are the components of the gas velocity vector. By the hodograph 'transformation,' the determination of the flow at a point in the physical (x,y) plane is made by examining the flow at a corresponding point of the hodograph plane.

Sonic Circle

The locus of those points in the hodograph plane where the gas velocity is equal to the local sound velocity. This locus is a circle.

Mach Number

A dimensionless parameter equal to the ratio of gas velocity to local sound velocity. In supersonic flow the Mach number is always greater than one. On the sonic circle the Mach number is one.

Mach Angle

The angle whose sine is the reciprocal of the Mach number. This angle is real for supersonic flows only.

Characteristics

A network of lines which is derived from the equations of motion and is used to characterize a particular solution. The analytic definition is given in Appendix A. In two dimensions there are two characteristics through each point. The flow quantities (pressure, density, velocity) are determined at any point by their values on those neighboring points which are on one of the two characteristics passing through the point in question, if the characteristics are real. This is the case in supersonic flow.

Rotationality and Vorticity

These quantities refer to the angular momentum present locally in a flow without singularities. The vorticity is proportional to the local angular momentum; when the vorticity is zero in a region, the flow throughout the region is said to be irrotational.

Shock Bottle

A configuration of shocks usually seen in high pressure jets, such as are found at the muzzles of guns. There are two shocks near the jet boundary, curving inward as one goes away from the exit of the nozzle, terminating in a Mach disk. This structure has the outlines of a bottle, hence the name. There are usually two shocks leaving the Mach disk which look like reflections of the two shocks forming the sides of the bottle. The two shocks nearest the nozzle exit are called intercepting shocks. Those on the downstream side of the Mach disk are called reflected shocks. (Figure 4).

Boundary Layer

A very thin region of transition in what is usually considered the boundary of the flow. The flow in this region is considered, in general, as being determined by the flow in the main portion of the stream, i.e., the flow in which no boundary layer is taken into account. The boundary layer is subsonic and often turbulent.

BASIC ASSUMPTIONS

The following assumptions are made:

The most general features of this type of flow are contained in the flow from a circular nozzle whose expansion ratio and divergence are small. This assumption is not questionable, except when one wishes to consider the flow on the axis near the nozzle. High pressure, in general, tends to decrease the dependence of the flow on nozzle shape at large distances and increase it at small distances from the exit section. The critical distance is of the order $\sqrt{M^2-1}\,L$, where L is the principal nozzle dimension and M is the Mach number at the exit section.

The flow is isentropic from the throat of the nozzle to the first shock reached in the atmosphere. This neglects the effects of minor shocks arising in the nozzle from surface defects, and shocks on the axis from lip effects, both of which must be neglected if the calculations are to be made in a reasonable time. Conversations with various authorities in the field reveal that the first of these effects is not likely to be large, while the last can not be estimated at this time.

The flow is irrotational; the ratio of specific heats is constant; secondary combustion can be neglected. These assumptions are necessary to facilitate computation. Irrotationality is associated with isentropic flow. Variations in the ratio of specific heats can be neglected, if previous work in the field is any indication. The neglect of secondary combustion is necessary in order to compute the points where secondary combustion can occur. In other words, such combustion is viewed as a small perturbation of the flow, like the boundary layer.

Boundary layer effects are to be excluded. This assumption is usually made in calculations like these, and indeed it is necessary to do so; but this question will be treated more fully in the body of the report.

Sufficient accuracy is obtained when the flow is determined via the method of characteristics for the corresponding two-dimensional case. This is standard procedure in computing axially symmetric flows. There are points where the correction is likely to be considerable; but in view of the various other approximations it is doubtful that the computations necessary to make this correction are justified at this time. The two-dimensional approximation will be treated more fully in the body of the report.

PROCEDURE

A two-dimensional, wedge shaped nozzle, whose expansion ratio is $\sqrt{2:1}$, and whose half-angle of divergence is 7°, is chosen. The chamber pressure is taken as 5400 psi. The chamber temperature is taken as 2400° K. Atmospheric pressure is taken as 14.7 psi, γ = ratio of specific heats = 1.4.

The method of characteristics is used to map the corresponding two-dimensional flow field. The characteristics method is adequately described in standard texts[1] and is reported more fully in Appendix A. Only a general description of the method will be given here.

The flow equations, which are discussed in Appendix A, are partial differential equations of the hyperbolic type (in the case of supersonic flow). The construction of the solution of a partial differential equation of hyperbolic type was formulated in great generality by Riemann around 1860. Earlier the method had been suggested for electrodynamics by Huyghens, on physical grounds. Thus, it is often the practice to refer to the Riemann method of solution as a Huyghens construction, even though the equation is not from electrodynamics.

In general, linear equations permit of much more elegant and sophisticated methods of solution, and the method of characteristics is reserved for such tasks as delimiting the region in which a solution exists. This is particularly the case when the coefficients are constant; but if the coefficients vary widely, and/or the equation is nonlinear, such methods fail and recourse must be had to the characteristics method.

The success of this method is limited to two dimensions, by and large, by computational difficulties. The accuracy is limited by the size of the mesh, and even though the initial steps are quite small, the mesh becomes very large if there is large expansion, and there is a resulting loss in accuracy. One must assume the state of flow along some initial curve. Boundary conditions, such as containment within walls and the existence of a free surface in the atmosphere, can be applied automatically. Other types of boundary conditions are difficult to apply but luckily do not often arise.

It should be understood that the method is valid only up to the formation of a shock, but it can be continued formally beyond the shock to give a lower order of approximation there.

Appendix A describes the analytical techniques for obtaining pressure and density from a map of the flow characteristics. The details of preparing such a map are given in Appendix B.

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[1]A. Ferri, "Elements of Aerodynamics of Supersonic Flows," Macmillan, New York, 1949; pp 19-40 (ch 2).

RESULTS AND DISCUSSION

Figure 1 is the map of the flow in the nozzle, and of the flow immediately beyond the nozzle exit. Figure 2 is the map of the entire flow up to the principal Mach disk. Figure 3 is a plot of the static pressure along the line of fire, behind the gun. Tables I and II enable one to find the pressure, density, velocity, etc., in any particular square of the flow map.

Flow in the Nozzle

Flow in the nozzle is almost identical with that predicted by hydraulic theory. This will not be the case in general for nozzles of much higher angles of divergence. The most striking feature of the flow immediately beyond the exit section is the sharp turn, through 57° (Figure 1), made by the boundary streamline in order to form a free surface at a pressure of 14.7 psi. From this point to the region of influence of the principal Mach disk the boundary curves back towards the axis.

Formation of the Shock Bottle

Rarefaction waves from the exit section are reflected from the boundary, and return to the main stream as compression waves which form an envelope called the intercepting shock (the sides of the shock bottle). Computation of the flow is continued formally in the region between this shock and the boundary. Neglect of shock conditions gives a lower order of accuracy there. This shock is comparatively weak, and corrections are not large, except in the neighborhood of the Mach disk and its reflected shock.

The inclination of the shock to the stream direction is almost constant at 24°. The rotationality introduced in the downstream flow depends on the entropy gradient there. The entropy gradient will be high, since the variation in Mach numbers upstream of the shock is from 6 to 10, roughly.

Pressures in Front of the Shock Bottle

Static pressure along the axis is easily plotted and the results are given in Figure 3. Units of length are arbitrary and are determined by the half height of the throat which is 1.25 units of length. The point of inflection corresponds to a focus in the actual three-dimensional case, where the sudden change in the streamline slope at the exit section makes itself felt as a logarithmic singularity at the indicated point on the axis. That point is the intersection of cone whose base is the exit section and whose apex angle is the exit Mach angle. This effect was discovered by S. A. Pai at the Institute for Fluid Dynamics, University of Maryland.

Pressure along other radial lines requires extensive interpolation before it can be tabulated. The very low pressures (and, consequently, temperatures) and very high Mach numbers produced in the flow should be noted. These point to the possibility of condensation and consequent condensation shocks.

Location of the Mach Disk

It was hoped that the method of calculation employed here would yield the location and extent of the principal Mach disk. It was surprising to find that this did not occur. The intercepting shock reflected from the symmetry axis and it was not possible to discern any reason for the location of a Mach disk. The shock pattern found was revealed to be well within critical limits and on the basis of this work alone there was no reason to expect a Mach disk.

Nevertheless, such disks have invariably been found in jets whose pressures, while more than a few atmospheres, were much lower than those used here. Moreover, the shock bottle, whose top is a Mach disk, has been observed time and again at the muzzles of guns. D. C. Pack of the Institute for Fluid Dynamics, University of Maryland, has said that similar difficulties had been noted by a number of his students. Consequently, it is believed that some other mechanism, possibly one heretofore purposely neglected, is responsible for this phenomenon. Courant and Friedrichs[2] express the opinion that the process of mixing with the surrounding air is largely the determining factor.

Some hypotheses about the formation of the Mach disk are:

1. The disk is a stable configuration established early in the formation of the jet by the action of transient phenomena.

2. In steady state flows, the transition of the boundary layer from laminar to turbulent flow causes a disturbance in the main stream which sets up a Mach disk.

In either case the mechanism is the same: vorticity is produced at some point or region and propagates into the main stream (via the heat-conduction-convection equation). Extra vorticity can be introduced into the flow only by curving the shocks more strongly. After a point an oblique shock is no longer possible and a normal shock (Mach disk) comes into being. Presumably, there is a point where no laminar flow is possible anywhere in the jet and the usual jet structure breaks down.

In the first case, vorticity is produced either by mixing with the air or interference from the nozzle lips. Then at some point in the early development of the jet, the vortex ring and the back pressure associated with the short length of the growing

[2]Courant and Friedricks, "Supersonic Flow," Interscience, New York, ~~1959~~ 1948, p. 392.

jet causes the flow to choose one possible pattern rather than another, the chosen pattern being stable.[3] Such ambiguities in supersonic flow are well known.

In the second case, the boundary layer changes abruptly from laminar to turbulent flow. In such a case the pressure on the main body of the stream changes sharply as the pressure gradient in the boundary layer changes from that characteristic of laminar flow to that characteristic of turbulence. Such a disturbance is propagated along a characteristic line to the rest of the flow. Such a disturbance can act not only as a source of vorticity but also as a strong pressure pulse. When this characteristic hits the intercepting shock, the curvature of the shock must change in order to take this new disturbance into account. If the pressure is sufficiently high (the critical value depends upon the nozzle) the shock must be normal. Unfortunately, the theory of the boundary layer transition is not formulated sufficiently at present for definite predictions to be made. Recent work on the transition from laminar to turbulent boundary layer flows shows that this transition is associated with Reynolds' numbers of the order 10^5 for conical flow (using velocity along the surface), 10^6 for flow over a flat plate. A quick, rough calculation shows that this magnitude Reynolds' number is likely to be reached at a point one-third of the way along the boundary of the jet between the exit section and the point of maximum expansion. This will reduce the distance of the Mach disk from the exit section to about one-third of the value shown in Figure 2.

It is reasonable to assume that the boundary layer transition will occur, if at all, between the lips of the nozzle and the point where the boundary becomes horizontal. The first point is the start of mixing with the surrounding atmosphere, the second the only other point distinguishable on physical grounds from any other point on the boundary. In other words, the assumption is made that the transition occurs in the first half of the main jet structure. This assumption is justified by Schlieren photographs of jets available from a number of sources.[4]

For the purposes of this analysis, the point of intersection of the characteristic from the horizontal part of the boundary with the intercepting shock was chosen as the location of the normal shock. This represents the maximum distance away from the nozzle of the Mach disk. This distance is of the order of 140 exit diameters. There are many indications that this is two or three times too large, indicating a transition point somewhat closer to the nozzle. Such an error is caused not only by improper location of the critical characteristic line, but also by neglect of any shocks or similar details on the axis caused by nozzle details. Otherwise, errors of this magnitude are not expected.

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[3]Hartmann, J. and Lazarus, F., "The Air-jet with a Velocity Exceeding That of Sound." Phil. Mag. 31, 7, 35-50, 1941.

[4]Hartmann and Lazarus, loc. cit.: Payman and Sheperd, "Explosion Waves and Shock Waves," Proc. Roy. Soc. A. 186, plate 17, Figures 10-20, (1946).

Experimental studies of the flow of powder gas through a conical nozzle similar in cross-section to the one studied here show that the Mach disk occurs about 15 exit diameters from the nozzle exit. This corresponds to the point (Figure 2) where the characteristic leaving from the focus intersects the boundary. As a number of authorities have mentioned, this corresponds to the first point where the entire flow is aware of the presence of the atmosphere. Hence, this represents the point closest to the nozzle that the Mach disk can occur. This can then be used as a criterion for locating the disk with considerably better accuracy than the method mentioned previously. These experimental studies were conducted some time after the main body of the report was written.

CONCLUSIONS

The study presented here represents an attempt to understand the most general features of the jet from a nozzle backed by very high pressure. As soon as a reliable method for locating the Mach disk is available, this study will provide an adequate representation of the flow in the section nearest the nozzle. In the meantime this study establishes the maximum extent of that flow.

The pressures indicated in the graph in Figure 3 and the corresponding dynamic pressures give a good indication of the forces to be expected in the region upstream of the Mach disk. In the downstream region an analysis of the mixing with the air is needed before similar conclusions can be drawn.

The extrapolation to three dimensions can be carried out formally by a method outlined in: R. Sauer, "Method of Characteristics for Three-Dimensional Axially Symmetric Super nic Flows," N.A.C.A. T.M. 1133, Washington, 1947. Examples given in this reference indicate that linear dimensions are decreased by a few per cent, in general. This arises from the extra room for expansion provided by the third degree of freedom. More quantitative statements than this can not be made at this time because of the difficulty in locating the principal Mach disk. The only effect opposing the decrease in linear dimensions is that of secondary combustion. The low temperatures and pressures before the disk and the high temperatures and pressures after the disk indicate that most secondary combustion will occur after the disk.

It should be pointed out that even a phenomenological theory for the location of the Mach disk is a new development in basic theory, and it is therefore recommended and requested that this work be pushed to its logical conclusion by both theoretical and experimental analyses.

FUTURE WORK

Future work on this particular phase of the blast program should be concentrated on experimental investigations designed to test the various hypotheses advanced as to the cause of the Mach disk. Two series of such experiments are planned:

A powder charge will be exploded in a suitably vented vessel and the development in time of the jet recorded with high speed spark or Schlieren photographs.

Different atmospheres will be introduced into a suitably chosen glass tube, and a jet formed by exhausting a high pressure (3000 psi) tank through a needle valve whose opening can be adiabatically controlled, will be studied.

The first of these experiments is in the process of development, while the second awaits adequate room and equipment.

Table I. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines

No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$
1	596	402	26	594	398	51	592	394	76	590	390
2	597	401	27	595	397	52	593	393	77	589.5	390
3	598	400	28	596	396	53	592	393	78	589	390.5
4	599	399	29	595	396	54	591	394	79	588.5	391
5	598	399	30	594	397	55	590	395	80	588	391.5
6	597	400	31	593	398	56	589	396	81	587.5	392
7	596	401	32	592	399	57	589	395	82	587	392.5
8	595	402	33	592	398	58	590	394	83	586.5	393
9	595	401	34	593	397	59	591	393	84	586.5	392.5
10	596	400	35	594	396	60	592	392	85	587	392
11	597	399	36	595	395	61	591	392	86	587.5	391.5
12	598	398	37	594	395	62	590	393	87	588	391
13	597	398	38	593	396	63	589	394	88	588.5	390.5
14	596	399	39	592	397	64	588	395	89	589	390
15	595	400	40	591	398	65	588	394	90	589.5	389.5
16	594	401	41	591	397	66	589	393	91	589	389.5
17	594	400	42	592	396	67	590	392	92	588.5	390.0
18	595	399	43	593	395	68	591	391	93	588	390.5
19	596	398	44	594	394	69	590	391	94	587.5	391
20	597	397	45	593	394	70	589	392	95	587	391.5
21	596	397	46	592	395	71	588	393	96	586.5	392
22	595	398	47	591	396	72	587	394	97	586.5	391.5
23	594	399	48	590	397	73	587	393	98	587	391
24	593	400	49	590	396	74	588	392	99	587.5	390.5
25	593	399	50	591	395	75	589	391	100	588	390

Table 1. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_{1/2}$	$C_{2/2}$	No. of Point	$C_{1/2}$	$C_{2/2}$	No. of Point	$C_{1/2}$	$C_{2/2}$	No. of Point	$C_{1/2}$	$C_{2/2}$
101	588.5	389.5	126	587.5	387.5	147	545	392.0	171	534	390.5
102	589	389	127	587	387.5	148	545	391.5	172	534.5	390.5
103	588.5	389	128	586.5	388	149	545	391.0	173	535	390.5
104	588	389.5	129	586.5	387.5	150	533	392.5	174	579	390
105	587.5	390	130	587	387	151	533	392.0	175	569.5	390
106	587	390.5	131	586.5	387	152	533	391.5	176	559	390
107	586.5	391	132	586.5	386.5	153	533	391.0	177	545	390
108	586.5	390.5	133-1	579	393	1stBP	533	393	178	533	390
109	587	390	133-2	569.5	393	154	533	391	179	533	390
110	587.5	389.5	133-3	559.0	393	155	533	392	180	535	390
111	588	389	133-4	545	393	156	533	391.5	181	534	390
112	588.5	388.5	133-5	533	393	157	533	391.0	182	534.5	390
113	588	388.5	134	579	392.5	158	535	392	183	535	390
114	587.5	389	135	579	392	159	535	391.5	184	535.5	390
115	587	389.5	136	579	391.5	160	534	391.5	185	579	389.5
116	586.5	390	137	579	391	161	535	391	186	569.5	"
117	586.5	389.5	138	569.5	392.5	162	534	391	187	559	"
118	587	389	139	569.5	392	163	534.5	391	188	545	"
119	587.5	388.5	140	569.5	391.5	164	579	390.5	189	533	"
120	588	388	141	569.5	391	165	569.5	390.5	190	533	"
121	587.5	388	142	559	392.5	166	559	390.5	191	535	"
122	587	388.5	143	559	392.0	167	545	390.5	192	534	"
123	586.5	389	144	559	391.5	168	533	390.5	193	534.5	"
124	586.5	388.5	145	559	391.0	169	533	390.5	194	535	"
125	587	388	146	545	392.5	170	535	390.5	195	535.5	"

Table 1. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$
196	536	389.5	221	537	388.5	246	536	387.5	271	533	386.5
197	579	389	222	579	388	247	536.5	"	272	533.5	"
198	569.5	"	223	569.5	"	248	537	"	273	534	"
199	559	"	224	559	"	249	537.5	"	274	534.5	"
200	545	"	225	545	"	250	538	"	275	535	"
201	533	"	226	533	"	251	579	387	276	538.5	"
202	533.5	"	227	533.5	"	252	569.5	"	277	536	"
203	534	"	228	534	"	253	559	"	278	536.5	"
204	534.5	"	229	534.5	"	254	545	"	279	537	"
205	535	"	230	535	"	255	533	"	280	537.5	"
206	535.5	"	231	535.5	"	256	533.5	"	281	538	"
207	536	"	232	536	"	257	534	"	282	538.5	"
208	536.5	"	233	536.5	"	258	534.5	"	283	539	"
209	579	388.5	234	537	"	259	535	"	284	585	385
210	569.5	"	235	537.5	"	260	535.5	"	288	579	"
211	559	"	236	579	387.5	261	536	"	289	569.5	"
212	545	"	237	569.5	"	262	536.5	"	290	559	"
213	533	"	238	559	"	263	537	"	291	545	"
214	533.5	"	239	545	"	264	537.5	"	292	533	"
215	534	"	240	533	"	265	538	"	293	533.5	"
216	534.5	"	241	533.5	"	266	538.5	"	294	534	"
217	535	"	242	534	"	267	579	386.5	295	534.5	"
218	535.5	"	243	534.5	"	268	569.5	"	296	535	385
219	536	"	244	535	"	269	559	"	297	535.5	"
220	536.5	"	245	535.5	"	270	545	"	298	536	"

Table I. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines (Cont'd)

No.of Point	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$
299	536.5	385	326	539.5	382	351	559	379	376	537.5	377.5
300	537	"	327	541	"	352	545	"	377	538	"
301	537.5	"	328	580.5	380.5	353	533	"	378	538.5	"
302	538	"	329	579	"	354	533.5	"	379	539	"
303	538.5	"	330	569.5	"	355	534	"	380	541	"
304	539	"	331	559	"	356	536	"	381	544	"
305	539.5	"	332	545	"	357	536.5	"	382	545.5	377.5
305.5	582	382	333	533	"	358	537	"	383	576	376
306	579	"	334	533.5	"	359	537.5	"	384	569.5	"
307	569.5	"	335	534	"	360	538	"	385	559	"
308	559	"	336	534.5	"	361	538.5	"	386	545	"
309	545	"	337	535.5	"	362	539	"	387	533	"
310	533	"	338	536	"	363	541	"	388	533.5	"
311	533.5	"	339	536.5	"	364	544	"	389	534	"
312	534	"	340	537	380.5	365	545.5	"	390	536	"
313	534.5	"	341	537.5	"	366	577.5	377.5	391	536.5	"
314	535	"	342	538	"	367	569.5	"	392	537	"
315	535.5	"	343	538.5	"	368	559	"	393	537.5	"
316	536	"	344	539	"	369	545	"	394	538	"
317	536.5	"	345	539.5	"	370	533	"	395	538.5	"
318	537	"	346	541	"	371	533.5	"	396	539	"
319	537.5	"	347	544	"	372	534	"	397	541	"
320	538	"	348	545.5	"	373	536	"	398	544	"
321	538.5	"	349	579	379	374	536.5	"	399	545.5	"
322	539	"	350	569.5	"	375	537	"	403	574.5	374.5

Table I. Correlation of Indices in Figures I and 2 with Numbers of Characteristic Lines (Cont'd)

No.of Poin.	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$	No.of Point	$C_1/2$	$C_2/2$
404	569.5	374.5	429	536	373	454	551.5	371	484	542	368
405	559	"	430	533	"	455	553	"	485	543	"
406	545	"	431	533.5	"	456	569.5	369.5	486	544	"
407	533	"	432	534	"	457	559	"	487	545.5	"
408	533.5	"	433	542	"	458	545	"	488	548.5	"
409	534	"	434	543	"	459	542	"	489	550	"
410	536	"	435	544	"	460	539	"	490	551.5	"
411	536.5	"	436	545.5	"	461	536	"	491	553	"
412	537	"	437	548.5	"	462	541	"	492	555	"
413	537.5	"	438	550	"	463	542	"	493	556.5	"
414	538	"	439	551.5	"	464	543	"	474	566.5	"
415	538.5	"	440	571	371	465	544	"	475	565	"
416	539	"	441	569.5	"	466	545.5	"	476	563.5	"
417	541	"	442	559	"	467	548.5	"	477	562	"
418	544	"	443	545	"	468	550	"	478	560.5	"
419	545.5	"	444	542	"	469	551.5	"	494	566.5	366.5
420	542	"	445	539	"	470	553	"	495	565	"
421	539	"	446	536	"	471	555	"	496	563.5	"
422	536	"	447	541	"	472	568	368	497	562	"
423	573	373	448	542	"	473	559	"	498	560.5	"
424	569.5	"	449	543	"	479	545	"	499	559	"
425	559	"	450	544	"	480	542	"	500	545	"
426	545	"	451	545.5	"	481	539	"	501	542	"
427	542	"	452	548.5	"	482	536	"	502	539	"
428	539	"	453	550	"	483	541	"	503	536	"

Table I. Correlation of Indices in Figures I and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$
504	542	366.5	529	553	365	555	552	362	580	550	360.5
505	544	"	530	555	"	556	551	"	581	549	"
506	545.5	"	531	556.5	"	557	550	"	582	548	"
507	548.5	"	532	558	"	558	549	"	583	547	"
508	550	"	533	559.5	"	559	548	"	584	546	"
509	551.5	"	535	563.5	363.5	560	547	"	585	545	"
510	553	"	536	562	"	561	546	"	586	544	"
511	555	"	537	560.5	"	562	545	"	587	543	"
512	556.5	"	538	559	"	563	544	"	588	542	"
513	558	"	539	552	"	564	543	"	589	541	"
514	565	365	540	545	"	565	542	"	590	559	359
515	563.5	"	541	542	"	566	541	"	591	558	"
516	562	"	542	539	"	567	540	"	592	557	"
517	560.5	"	543	545.5	"	568	539	"	593	556	"
518	559	"	544	548.5	"	569	550	"	594	555	"
519	545	"	545	550	"	570	560.5	360.5	595	554	"
520	542	"	546	562	362	571	559	"	596	553	"
521	539	"	547	560.5	"	572	558	"	597	552	"
522	536	"	548	559	"	573	557	"	598	551	"
523	542	"	549	558	"	574	556	"	599	550	"
524	544	"	550	557	"	575	555	"	600	549	"
525	545.5	"	551	556	"	576	554	"	601	548	"
526	548.5	"	552	555	"	577	553	"	602	547	"
527	550	"	553	554	"	578	552	"	603	546	"
528	551.5	"	554	553	"	579	551	"	604	545	"

Table I. Correlation of Indices in Figures I and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$
605	544	359	630	551	357	655	553	355	681	546	356
606	543	"	631	550	"	656	552	"	682	541	359
607	542	"	632	549	"	657	551	"	683	540	360.5
608	558	358	633	548	"	658	550	"	684	539	"
609	557	"	634	547	"	659	549	"	685	545	356
610	556	"	635	546	"	660	548	"	686	544	"
611	555	"	636	545	"	661	547	"	687A	543	"
612	554	"	637	544	"	662	554	354	687	542	358
613	553	"	638	543	"	663	553	"	688	546	355
614	552	"	639		"	664	552	"	689	545	"
615	551	"	640		"	665	551	"	690	552	352
616	550	"	641		"	666	550	"	691	551	"
617	549	"	642		"	667	549	"	692	550	"
618	548	"	643	556	356	668	548	"	693	549	"
619	547	"	644	555	"	669	547	"	694	548	"
620	546	"	645	554	"	670	546	"	695	547	"
621	545	"	646	553	"	671	545	"	696	551	351
622	544	"	647	552	"	672	553	353	697	550	"
623	543	"	648	551	"	673	552	"	698	549	"
624	557	357	649	550	"	674	551	"	699	550	350
625	556	"	650	549	"	675	550	"	700	549	"
626	555	"	651	548	"	676	549	"	703	551.5	363.5
627	554	"	652	547	"	677	548	"	704	553	"
628	553	"	653	555	355	678	547	"	505	555	"
629	552	"	654	554	"	679			706	556.5	"

Table I. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$	No. of Point	$C_1/2$	$C_2/2$
707	558	363.5	732	558	359	747	568	357	772	565.5	354
708	559.5	"	733	559.5	"	748	569	"	773	567	"
709	561	"	734	561	"	749	556.5	356	774	568	"
710	551.5	362	735	562.5	"	750	558	"	775	569	"
711	553	"	736	564	"	751	559.5	"	776	570	"
712	555	"	737	565.5	"	752	561	"	777	571	"
713	556.5	"	738	567	"	753	562.5	"	678	547	353
714	558	"	688A	556.5	358	754	564	'	778	546	"
715	559.5	"	689A	558	"	755	565.5	"	779	545	"
716	561	"	690A	559.5	"	756	567	"	780	561	"
717	562.5	"	691A	561	"	757	568	"	781	562.5	"
718	551.5	360.5	692A	562.5	"	758	569	"	782	565.5	"
719	553	"	693A	564	"	759	544	355	783	567	"
720	555	"	694A	565.5	"	760	559.5	"	784	568	"
721	556.5	'	695A	567	"	761	561	"	785	546	352
722	558	"	696A	568	"	762	562.5	"	786	545	"
723	559.5	"	639	542	357	763	564	"	787	548	351
724	561	"	739	556.5	"	764	565.5	"	788	547	"
725	562.5	"	740	558	"	765	567	"	789	546	"
726	564	"	741	559.5	"	766	568	"	790	545 561	" "
727	565.5	"	742	561	"	767	569	"	791	549	350
728	551.5	359	743	562.5	"	768	545	354	792	548	"
729	553	"	744	564	"	769	561	"	793	547	"
730	555	"	745	565.5	"	770	562.5	"	794	546	"
731	556.5	"	746	567	"	771	564	"	795	545	"

Table 1. Correlation of Indices in Figures 1 and 2 with Numbers of Characteristic Lines (Cont'd)

No. of Point	$C_1/2$	$C_2/2$	*No. of Point*	$C_1/2$	$C_2/2$	*No. of Point*	$C_1/2$	$C_2/2$	*No. of Point*	$C_1/2$	$C_2/2$
796	544	350	807	562.5	352	818	568	351	828	569	350
797	543	"	808	565.5	"	819	569	"	829	570	"
798	542	"	809	567	"	820	570	"	830	547 561	349
799	549	349	810	568	"	821	571	"	831	565.5	"
800	548	"	811	569	"	822	572	"	832	567	"
801	570	355	812	570	"	823	561	350	833	568	"
802	571	"	813	571	"	824	562.5	"	834	569	"
803	569	353	814	572	"	825	565.5	"	835	570	"
804	570	"	815	562.5	351	826	567	"	836	548	348
805	571	"	816	565.5	"	827	568	"	837	567	365.5
806	572	"	817	567	"						

Table II. Numerical Relations for the Two-dimensional Characteristics System

The table gives as a function of the characteristic number N (column 1) the following quantities:

Column 2 The value of ν, deviation of expansion from Mach number 1.

Column 3 The value of the stream Mach number (determined from column 2).

Column 4 The ratio of the local static pressure p to the critical static pressure p^* corresponding to the pressure for ($M = 1$).

Column 5 The Mach angle μ determined from column 3.

Note: $\frac{C_1 + C_2}{2} = N$

$N = \frac{C_1 + C_2}{2}$	ν	M	$\frac{p}{p^*}$	μ	N	ν	M	$\frac{p}{p^*}$	μ
1000	0°	1.0000	1.00000	90°	963	37°	2.4107	.12735	24°30′
999	1°	1.0813	.90724	67°37′	962	38°	2.4525	.11970	24° 4′
998	2°	1.1327	.85100	61°59′	961	39°	2.4946	.11177	23°37′
997	3°	1.1764	.80486	58°13′	960	40°	2.5373	.10454	23°12′
996	4°	1.2185	.76192	55° 9′	959	41°	2.5812	.09770	22°48′
995	5°	1.2577	.72329	52°39′	958	42°	2.6256	.09117	22°23′
994	6°	1.2938	.68896	50°37′	957	43°	2.6712	.08500	21°59′
993	7°	1.3298	.65588	48°45′	956	44°	2.7175	.07917	21°36′
992	8°	1.3648	.62488	47° 6′	955	45°	2.7650	.07359	21°13′
991	9°	1.4000	.59484	45°35′	954	46°	2.8118	.06848	20°50′
990	10°	1.4350	.56608	44°10′	953	47°	2.8601	.06363	20°28′
989	11°	1.4690	.53921	42°54′	952	48°	2.9112	.05895	20° 5′
988	12°	1.5032	.51325	41°42′	951	49°	2.9613	.05466	19°44′
987	13°	1.5368	.48875	40°36′	950	50°	3.0122	.05065	19°23′
986	14°	1.5706	.46512	39°33′	949	51°	3.0650	.04660	19° 2′
985	15°	1.6046	.44233	38°33′	948	52°	3.1188	.04310	18°42′
984	16°	1.6382	.42075	37°37′	947	53°	3.1735	.03985	18°22′
983	17°	1.6721	.39995	36°44′	946	54°	3.2263	.03669	18° 3′

Table II. Numerical Relations for the Two-dimensional Characteristics System (Cont'd)

$N = \frac{C_1 + C_2}{2}$	ν	M	$\frac{p}{p^*}$	α	N	ν	M	$\frac{p}{p^*}$	α
982	18°	1.7064	.37981	35°53′	945	55°	3.2866	.03375	17°43′
981	19°	1.7408	.36053	35° 4′	944	56°	3.3455	.03103	17°24′
980	20°	1.7753	.34210	34°16′	943	57°	3.4065	.02843	17° 4′
979	21°	1.8098	.32454	33°32′	942	58°	3.4662	.02605	16°45′
978	22°	1.8446	.30768	32°49′	941	59°	3.5300	.02382	16°27′
977	23°	1.8796	.29152	32° 8′	940	60°	3.5945	.02175	16° 9′
976	24°	1.9148	.27610	31°29′	939	61°	3.6605	.01984	15°51′
975	25°	1.9503	.26136	30°50′	938	62°	3.7283	.01838	15°33′
974	26°	1.9860	.24727	30°14′	937	63°	3.7975	.01643	15°16′
973	27°	2.0235	.23324	29°37′	936	64°	3.8699	.01489	14°58′
972	28°	2.0600	.22034	29° 2′	935	65°	3.9425	.01350	14°42′
971	29°	2.0968	.20805	28°29′	934	66°	4.0175	.01223	14°25′
970	30°	2.1320	.19690	27°58′	933	67°	4.0950	.01103	14° 8′
969	31°	2.1726	.18483	27°24′	932	68°	4.175	.00990	13°56′
968	32°	2.2105	.17430	26°54′	931	69°	4.256	.00889	13°35′
967	33°	2.2485	.16400	26°24′	930	70°	4.340	.00800	13°19′
966	34°	2.2879	.15406	25°55′	929	71°	4.427	.00720	13° 3′
965	35°	2.3290	.14480	25°25′	928	72°	4.517	.00646	12°47′
964	36°	2.3693	.13582	24°58′	927	73°	4.610	.00577	12°32′
926	74°	4.704	.00510	12°16′	909	91°	7.006	.000455	8° 7′
925	75°	4.802	.00450	12° 1′	908	92°	7.202	.000381	7°54′
924	76°	4.905	.00400	11°46′	907	93°	7.407	.000318	7°41′
923	77°	5.010	.00354	11°31′	906	94°	7.622	.000265	7°28′
922	78°	5.119	.00313	11°16′	905	95°	7.853	.000220	7°15′

Table II. Numerical Relations for the Two-dimensional Characteristics System (Cont'd)

$N = \frac{C_1 + C_2}{2}$	ν	M	$\frac{p}{p^*}$	μ	N	ν	M	$\frac{p}{p^*}$	μ
921	79°	5.231	.00277	11° 1′	904	96°	8.092	.000182	7° 3′
920	80°	5.348	.002413	10°47′	903	97°	8.343	.000149	6°50′
919	81°	5.472	.002106	10°32′	902	98°	8.616	.0001194	6°37′
918	82°	5.599	.001835	10°17′	901	99°	8.902	.0000965	6°25′
917	83°	5.730	.001598	10° 3′	900	100°	9.210	.0000770	6°12′
916	84°	5.865	.001383	9°41′	899	101°	9.535	.0000614	5°59′
915	85°	6.007	.001193	9°27′	898	102°	9.881	.0000481	5°47′
914	86°	6.152	.001023	9°14′	897	103°	10.260	.0000377	5°34′
913	87°	6.307	.000879	9° 1′	896	104°	10.665	.0000294	5°21′
912	88°	6.472	.000751	8°47′	895	105°	11.088	.0000224	5° 9′
911	89°	6.642	.000638	8°34′	894	106°	11.552	.0000168	4°57′
910	90°	6.821	.000540	8°20′					

APPENDIX A

The Method of Characteristics for two-dimensional, steady, isentropic flow in two dimensions.

I. Theory

Let u and v be the x and y components of velocity q, respectively, and let θ be the angle the vector velocity $\bar{q}$ makes with some fixed reference line in the x, y plane. Let p be the pressure, C_p = specific heat at constant pressure, C_v = specific heat at constant volume, r the density, k = the ratio of specific heats $\frac{C_p}{C_v}$. The differential equations of motion for steady, perfect, irrotational, isentropic flow in two dimensions are: (see, e. g. Courant and Friedrichs, Loc. Cit.)

$$uu_x + vu_y = \frac{-1}{r}(p_x) \tag{1}$$

$$uv_x + vv_y = \frac{-1}{r}(p_y)$$

$$(ru)_x + (rv)_y = 0 \tag{2}$$

$$v_x - u_y = 0 \tag{3}$$

$$p = p(r) \text{ only}$$

$$p = ar^k$$

$$a = \text{const.}$$

From the sound speed c, $c^2 = \frac{kp}{r}$, we have $r_x = \frac{1}{c^2} p_x$. This together with (1) and (2) gives

$$(u^2 - c^2)u_x + uv(u_y + v_x) + (v^2 - c^2)v_y = 0 \tag{4}$$

The relation (3) implies the existence of a function φ (x,y), called the potential function, which satisfies

$$u = \varphi_x, \quad v = \varphi_y$$

In (4) this gives:

$$(u^2 - c^2)\varphi_{xx} + 2uv\varphi_{xy} + (v^2 - c^2)\varphi_{yy} = 0, \tag{5}$$

a nonlinear differential equation for the determination of the potential φ (x,y). It is well known that such a partial differential equation defines a family of curves S(x,y) = Const., called the characteristics. They are obtained by the substitution

$S_x^2 = \varphi_{xx}$, $S_y^2 = \varphi_{yy}$, $S_xS_y = \varphi_{xy}$. Then from (5):

$$(u^2 - c^2)\, S_x^2 + 2uvS_xS_y + (v^2 - c^2)S_y^2 = 0 \tag{6}$$

The slope of a surface $S(x,y)$ is given by $m = \frac{dy}{dx} = \frac{-S_x}{S_y}$ so that (6) gives $(u^2 - v^2)m^2 - 2uvm\,(v^2 - c^2) = 0$. This quadratic equation has the two solutions

$$\left.\begin{matrix} m+ \\ m- \end{matrix}\right\} = \frac{2uv \pm \sqrt{4u^2v^2 - 4(u^2 - c^2)\,(v^2 - c^2)}}{2(u^2 - v^2)}$$

Evidently, at any point in the physical (x,y) plane there are defined two curves, depending on velocity solutions u,v of equation (5). These slopes are real if and only if

$$4u^2v^2 - 4(u^2 - c^2)\,(v^2 - c^2) > 0$$

$$\text{i.e., } u^2 + v^2 = q^2 > c^2 \tag{7}$$

the flow is supersonic. If the above quantity (7) is equal to 0 at a point, the flow is sonic there, and there is only one characteristic at the point. Evidently the net formed by the two characteristics through each point is degenerate on a sonic line in the flow.

The characteristics can be shown to have the following properties:

1. Discontinuities in the derivatives of u and v can only occur on these characteristic, or "Mach", lines. Thus the Mach lines can be visualized as wave fronts in supersonic flow.

2. The characteristics at a point make equal angles A with the streamline, and this angle is the so-called Mach angle. We find

$$\sin A - \frac{c}{q}$$

3. The characteristic net represents a coordinate system in the physical plane (except on degenerate sonic lines). If we introduce parameters a, b along the Mach lines, we have the characteristics given by

$$C^+: \quad y_a = m_+x_a$$

$$C^-: \quad y_b = m\text{-}x_b$$

We may interchange dependent and independent variables in equation (4) by the transformation

$$u_x = jy_v,\ uy = jx_v,\ v_x = jy_u,\ v_y = jx_u$$

as long as the Jacobian determinant $j = u_x v_y - u_y v_x$ does not vanish in the region under consideration. We find, analogous to equations (3) and (4),

$$\begin{aligned} &(a) \quad x_v - y_u = 0 \\ &(b) \quad (u^2 - c^2)y_v - uv(x_v + y_u) + (v^2 - C^2)x_u = 0 \end{aligned} \tag{8}$$

By the first, there exists a function (u,v), a "hodograph potential," which satisfies

$$(u^2 - c^2)\,\Phi_{vv} - 2uv\,\Phi_{uv} + (v^2 - c^2)\,\Phi_{uu} - 0 \tag{9}$$

Φ (u,v) is the Legendre (contact) transformation of $\varphi(x,y)$: $\Phi = ux + vy - \varphi$. Note that this equation is linear. A new set of characteristics T(u,v) is found as before to satisfy:

$$(u^2 - c^2)T_v^{\ 2} - 2uvT_uT_v + (v^2 - c^2)T_u^{\ 2} = 0$$

The slope of these u, v characteristics is found to be $\dfrac{du}{dv} = \dfrac{-m_-}{-m_+}$, or

$$\begin{aligned} G^+&: \quad u_a = -m_- v_a \\ G^-&: \quad u_b = -m_+ v_b \end{aligned} \tag{10}$$

1. These characteristics are <u>independent</u> of the solution x = x(u,v), y = y(u,v), and are thus fixed in the u,v plane.

2. The relations $u_a x_b + v_a y_b = 0$ can be obtained, and indicate the following

$$u_b x_a + v_b y_a = 0$$

vital fact: Consider the image point u,v of a physical point x,y ($j(x,y) = 0$). The C^+ characteristic through x,y is perpendicular to the C^- characteristic through u,v, and the C^- characteristic through x,y is perpendicular to the G^+ characteristic through u,v. This is the basis of the graphical construction method. Equations (10) are integrated directly using Bernoulli's equation for c:

$$\mu^2 q^2 + (1 - \mu^2)c^2 = c_*^{\ 2},\quad \mu^2 = \frac{k - 1}{k + 1}$$

One finds the characteristics:

$$\mp 2\theta = \sqrt{\frac{k+1}{k-1}}\ \mathrm{Sin}^{-1}\left[(k-1)\frac{q^2}{c*^2} - k\right] \tag{11}$$

$$+ \mathrm{Sin}^{-1}\left[(k+1)\frac{c*^2}{q^2} - k\right] + 2c$$

where $q = c_*$ at $M = 1$. These curves are epicycloids, and are generated by the motion of a rim point of a circle of diameter $c_* \ (\frac{1}{u} - 1)$ rolling on a circle of radius c_*. The pressure ratio is related to the velocity by

$$\frac{q^2}{c_*{}^2} = (1 - p/p_o)^{\frac{k-1}{k}} \left(\frac{k+1}{k-1}\right) \tag{12}$$

where p_o is the stagnation pressure (chamber pressure). Thus the pressure is constant on circles concentric with the sonic circle in the u,v plane.

For practical use, it is better to consider θ positive. This can be done by taking θ = 200 degrees for an undisturbed flow, v = 0. (See p 36, A. Ferri.) Every characteristic can then be distinguished by the value of the constant C in equation (11). For the G^+ characteristic,

$$-\theta = f(q) - c_1 \tag{13}$$

where C_1 is the absolute magnitude of the constant that in equation (11) is negative. For the G^- characteristic,

$$\theta = f(q) - c_2; \tag{14}$$

eliminating f, then θ in (13) and (14), we have

$$\theta = \frac{c_1 - c_2}{2} \tag{15}$$

$$f(q) = \frac{c_1 + c_2}{2} = N$$

From (12) and (15), $f(q)$ is determined by $\frac{c_1 + c_2}{2}$ and the corresponding values of $\frac{p}{p^*}$, the Mach number M, the Mach angle, and the angle for the corresponding Prandtl-Meyer flow are given in Table II. Thus, entering in Table VI the values of $\frac{c_1 + c_2}{2}$ for points in the flow, taken from Table I, yields the values of the flow quantities at those points.

II. Application of method of characteristics to the construction of the flow through a nozzle.

Given a nozzle symmetric about the center line (which is chosen to be the x-axis), locate the throat of the nozzle at the y-axis. Now plot the nozzle contour so that it is congruent with the actual contour but of convenient size. The hodograph is now mounted so that the line whose direction number is 200 is paralled to the x-axis.

Since the Mach net is degenerate on the sonic line $x = 0$, choose the initial curve in this way: Through that point where the nozzle has at least a slope of 1° (or a multiple thereof) pass a circular arc whose center is the intersection of the tangent to the nozzle contour at that point with the x-axis. By the hydraulic theory, compute the flow quantities on this arc. From Table II, find the pressure number, N. Divide the arc into equal sub-arcs whose included angle is 1° (or a multiple thereof). At the end point of each arc label the point in a suitable fashion. The pressure number corresponding to each point is N. The direction number is 200 plus the included angle (in degrees) with the x-axis. The two characteristics in the hodograph plane which pass through the image of each point can now be deduced. Two adjacent points in the x,y plane determine a third point as follows:

Let the upper point be A, the lower one, B. Corresponding to the upward-going G^+ through the image of A in the hodograph plane, there is a downward sloping Mach line (G^-) through A, perpendicular to the G^+ characteristic. Similarly, there is a line C^+, through B, sloping upwards, perpendicular to the G^- (downward sloping) characteristic through the image of B in the hodograph plane. The intersection of the C^- line from A and the C^+ line from B determines a third point, P, where the flow quantities are determined by the location of the intersection of the corresponding G^+ and G^- characteristics in the hodograph plane. This process may now be continued.

At boundaries, if the boundary is a rigid wall, the new point is determined by one characteristic and the fact that the direction number must correspond to the slope of the wall. At a free surface, the new point is determined by one characteristic and the requirement that the pressure index, N, has a certain value which corresponds to receiver pressure.

III. Construction of the Hodograph

The equipment usually used includes a hodograph with a suitable ellipse and a drafting tool. Their use is outlined in I. From the knowledge of the pressure and direction of flow at a point, one can, using the hodograph, determine the Mach angle and the directions of the characteristics passing through that point. Or, knowing which two characteristics pass through a point in space, one can find the flow quantities there by using the hodograph.

In order to fix the quantities involved in such calculations, it is necessary to index the characteristics in a standard fashion. That method of indexing used here is the one proposed by Buseman and is in general use. The procedure is as follows:

1. Choose a radius for the sonic circle.

2. The hodograph can now be drawn according to the directions given in I.

3. Draw one characteristic leaving the sonic circle every two degrees, curving downward, and another set, leaving from the same points, curving upward.

4. Number the lines in each set so that at any point along the sonic circle the sum (a) of the number given one line, and (b) of the number from the line at that point which belongs to the other family is 1000.

5. At that point which is the intersection of the sonic circle with the radius which is paralleled to the arbitrarily chosen fixed line in space, make the difference of the numbers referred to in (4) be 200. Then the sum (called the pressure index) of the numbers of the two characteristics through some point in space gives the ratio: Critical speed/flow speed. (This relationship is shown in Table II.) The difference of the two numbers gives the direction of the flow with respect to the axis fixed in space. From the former number other pertinent quantities can be obtained (Table II).

The initial curve is taken to be the arc of the circle whose center is the vertex of the nozzle and which passes through the endpoints of the throat section. On that line it is assumed that the flow velocity-local sound velocity (equals throat, or critical, velocity) and the pressure index is then 1000 there. The flow is now determined by fixing the atmospheric pressure, which is assumed to be 14.7 psi.

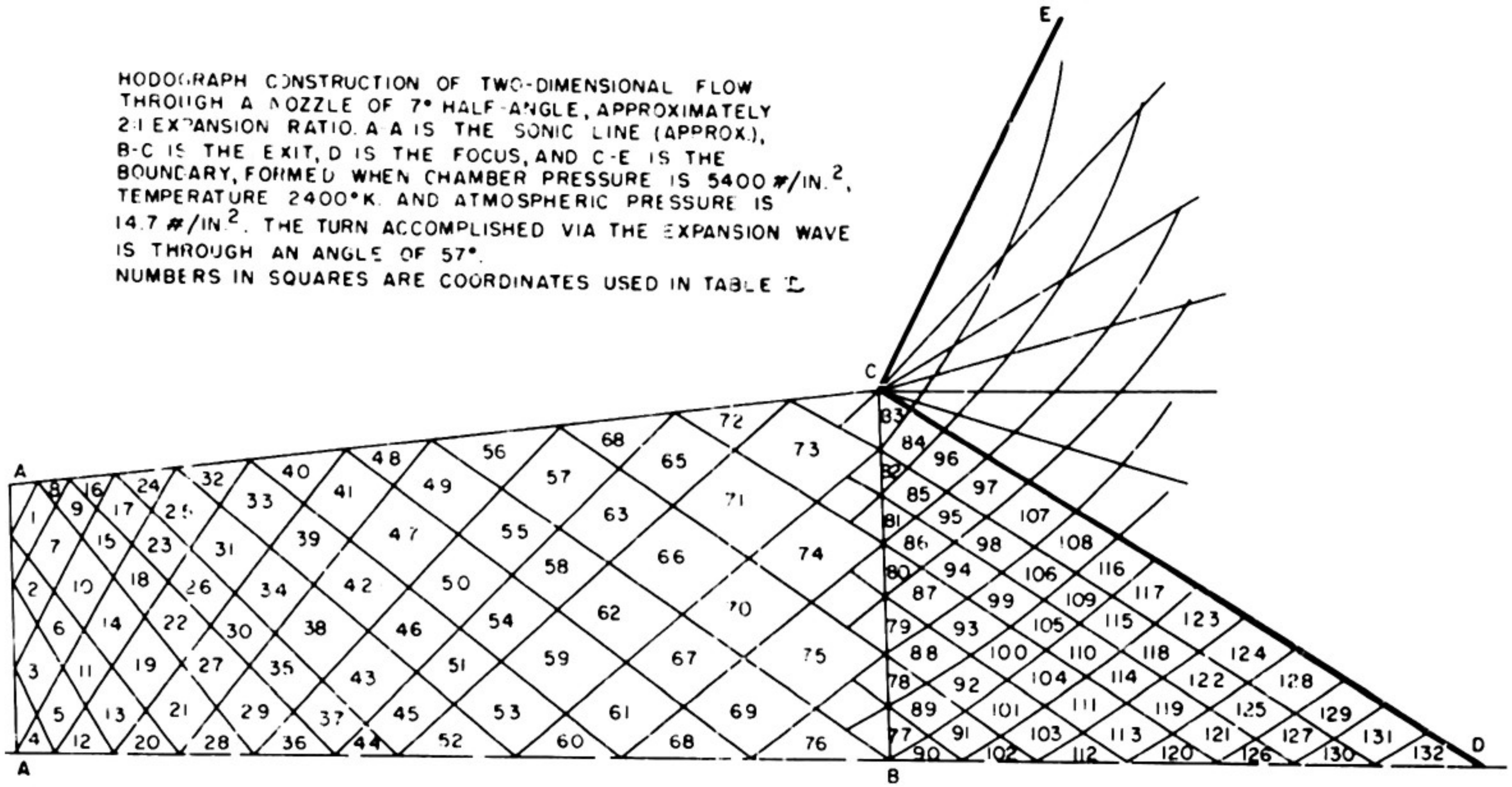


Figure 1. Flow through a nozzle of 7° half-angle

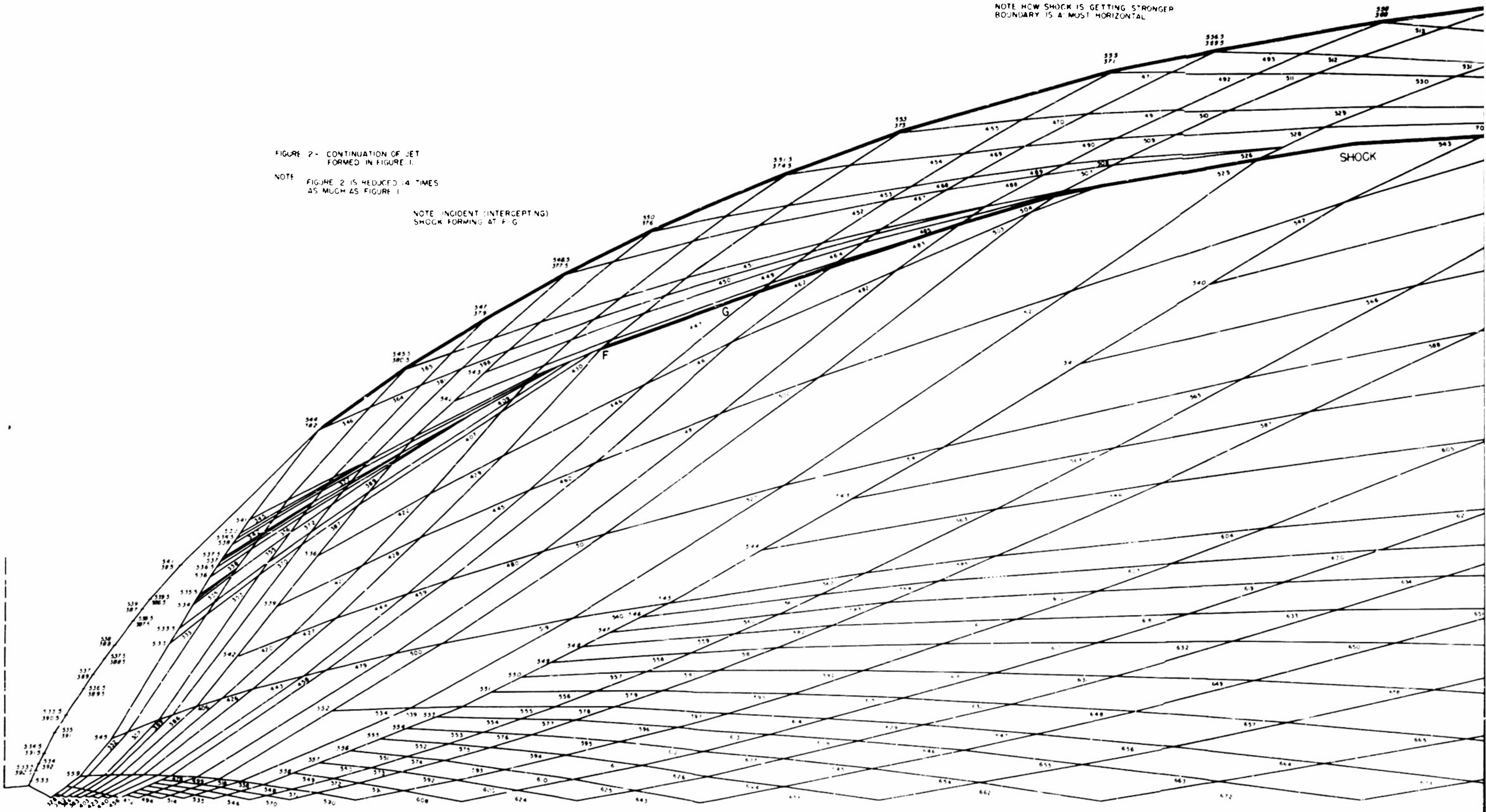
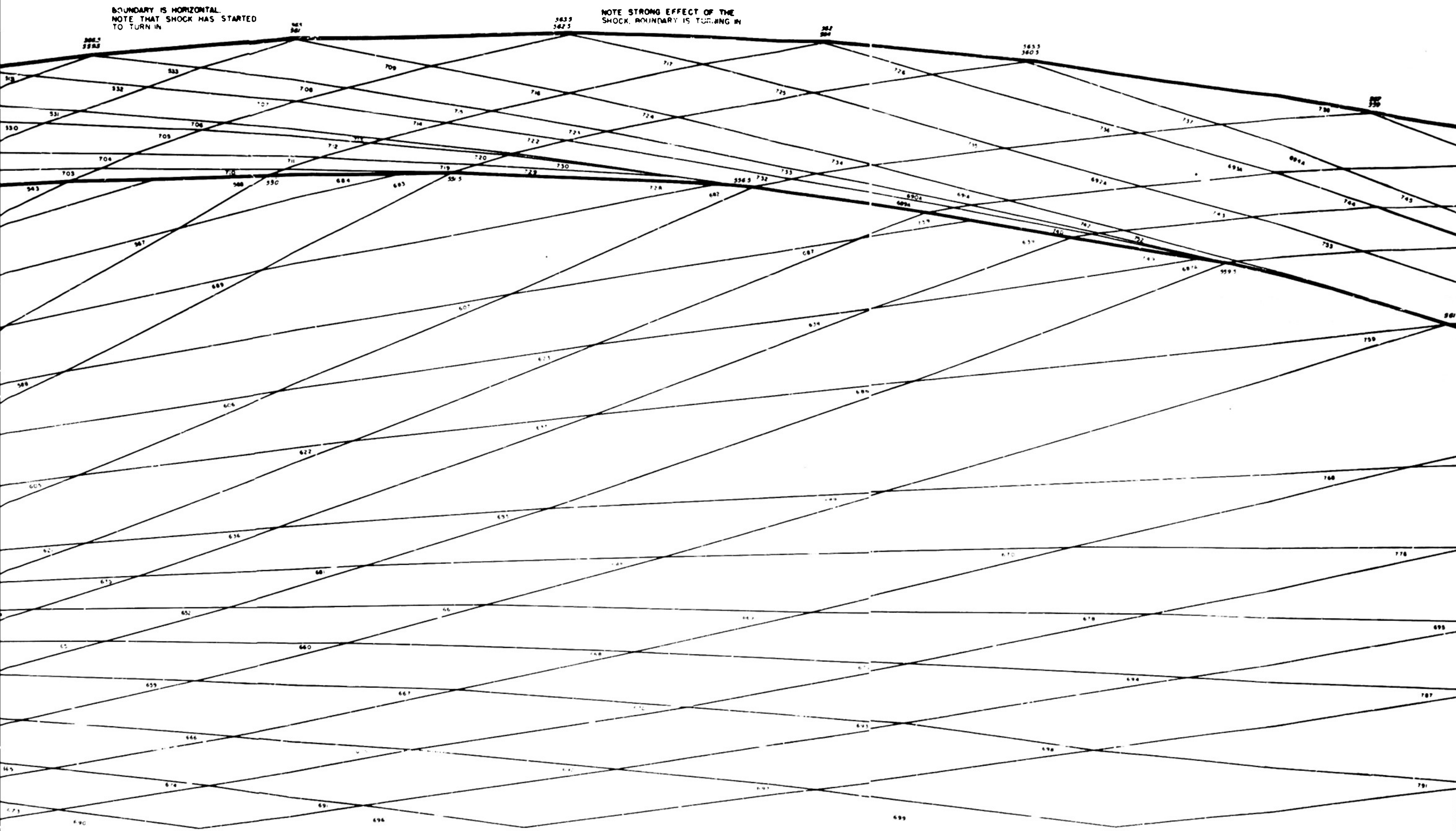
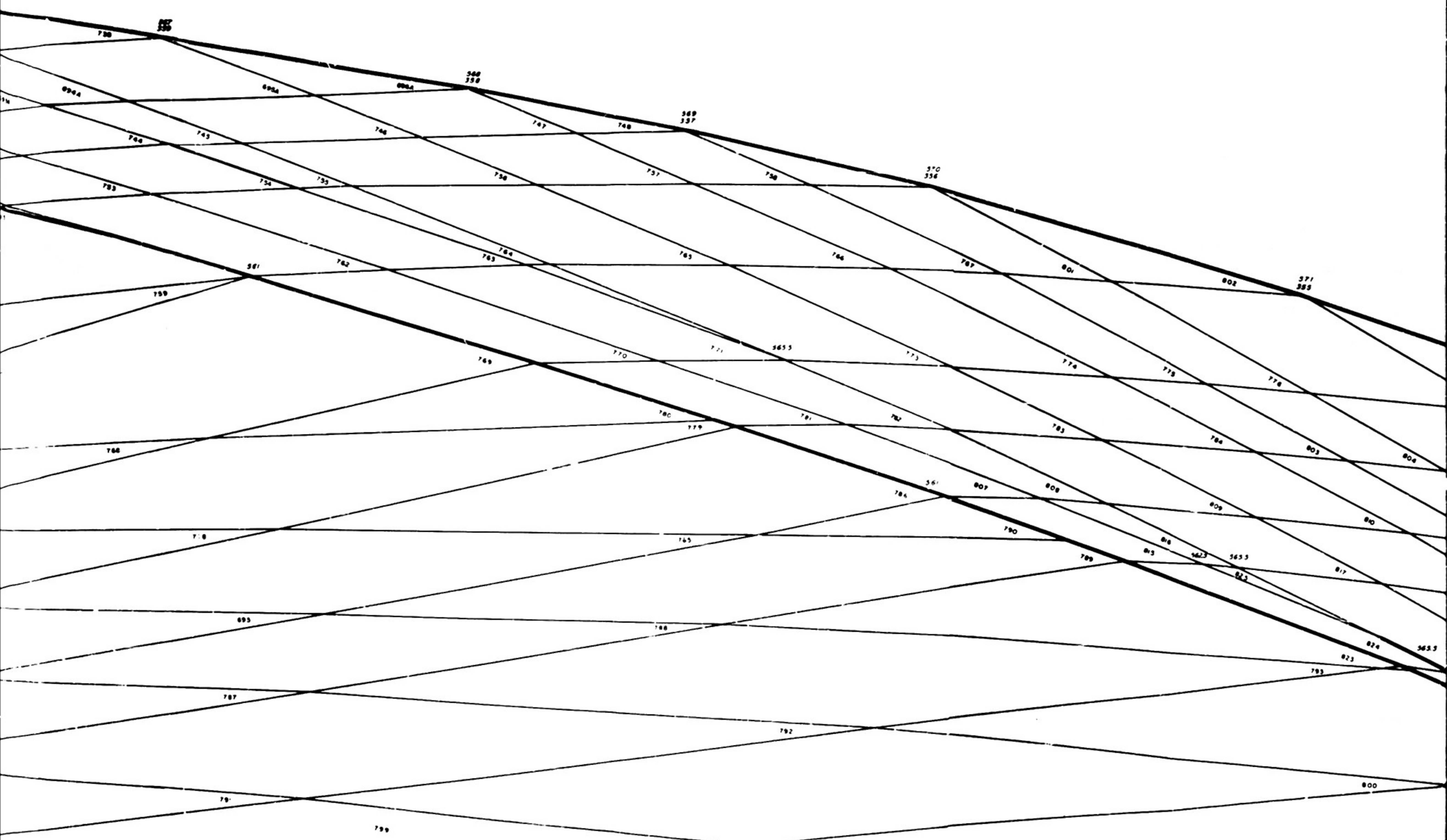


FIGURE 2 - CONTINUATION OF JET FORMED IN FIGURE 1.

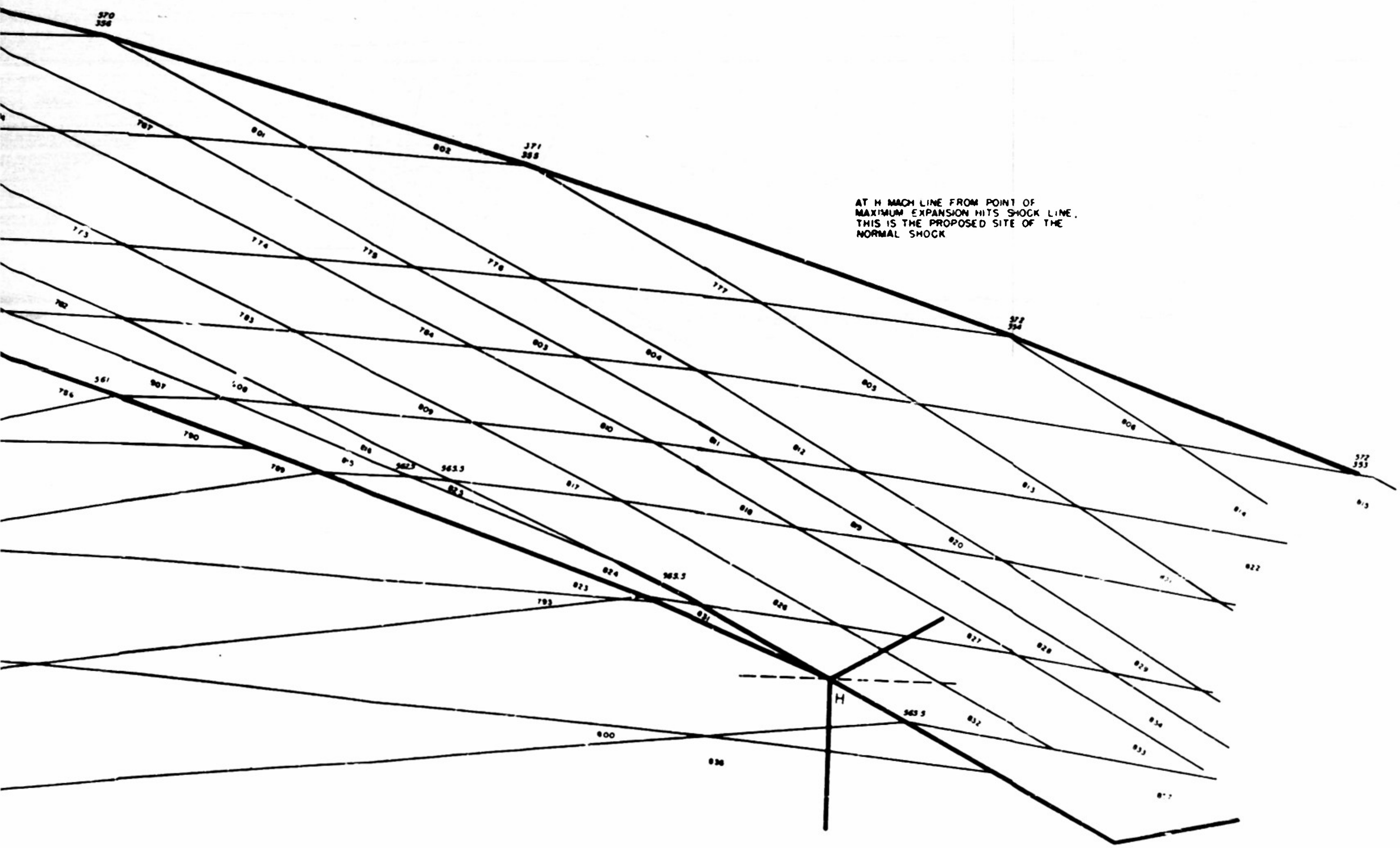

BOUNDARY IS HORIZONTAL.
NOTE THAT SHOCK HAS STARTED
TO TURN IN
NOTE STRONG EFFECT OF THE
SHOCK. BOUNDARY IS TURNING IN

2



3

Neg.#23358-2
R-1107



4

Neg.#23358-3
R-1107

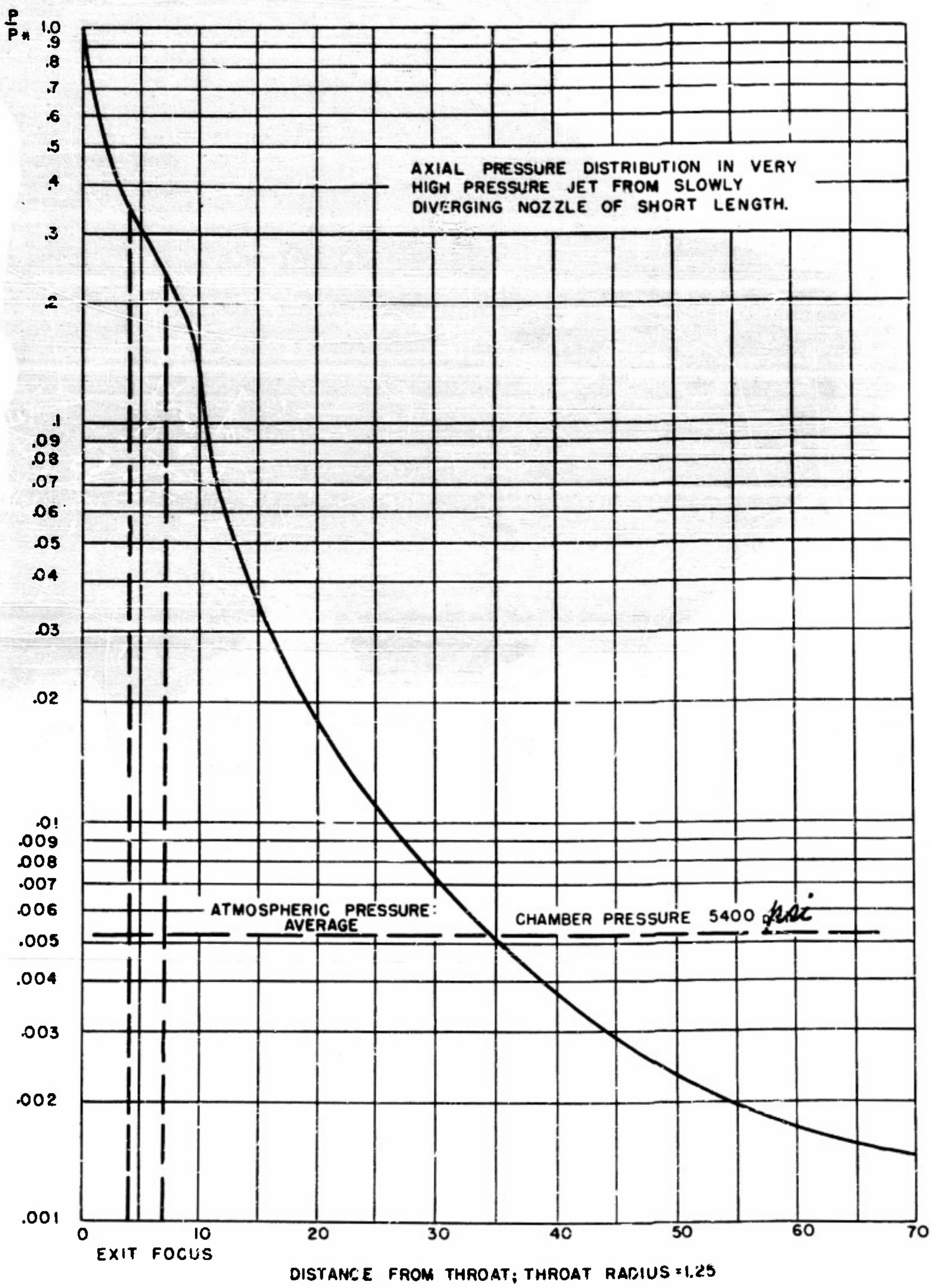


Figure 3. Pressure along the axis vs distance from throat

Neg.#23358-4
R-1107

Figure 4. A typical shock bottle formed at a gun muzzle